

**Thurlaston Church of England (Aided) Primary School**



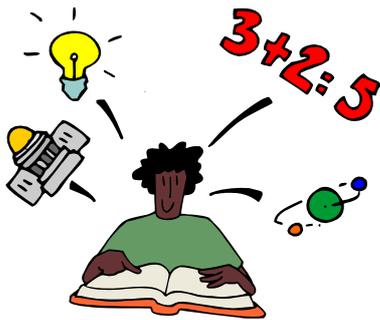
# **Maths Calculation Policy**

Adopted on 6<sup>th</sup> March 2008

Signed .....

# Written Calculation Policy Key Stages 1 - 3 LA Recommendations

Leicestershire Mathematics  
Team  
December 2007



Review date: August 2009  
at schools write a background to  
. It should contain some of the

\_\_\_\_\_ School  
Pencil and paper procedures

## **Background to the policy**

This policy contains the key pencil and paper procedures that will be taught within our school. It has been written to ensure consistency and progression throughout the school and reflects a whole school agreement.

**Although the focus of the policy is on pencil and paper procedures it is important to recognise that the ability to calculate mentally lies at the heart of the Primary National Strategy for mathematics. The mental methods in the *Primary Framework for teaching mathematics* will be taught systematically from Reception onwards and pupils will be given regular opportunities to develop the necessary skills. However mental calculation is not at the exclusion of written recording and should be seen as complementary to and not as separate from it. In every written method there is an element of mental processing. Sharing written methods with the teacher encourages children to think about the mental strategies that underpin them and to develop new ideas. Therefore written recording both helps children to clarify their thinking and supports and extends the development of more fluent and sophisticated mental strategies.**

During their time at this school children will be encouraged to see mathematics as both a written and spoken language. Teachers will support and guide children through the following important stages:

- developing the use of pictures and a mixture of words and symbols to represent numerical activities;
- using standard symbols and conventions;
- use of jottings to aid a mental strategy;
- use of pencil and paper procedures;
- use of a calculator.

**This policy concentrates on the introduction of standard symbols, the use of the empty number line as a jotting to aid mental calculation and on the introduction of pencil and paper procedures. It is important that children do not abandon jottings and mental methods once pencil and paper procedures are introduced. Therefore children will always be encouraged to look at a calculation/problem and then decide which is the best method to choose – pictures, mental calculation with or without jottings, structured recording or a calculator. Our long-term aim is for children to be able to select an efficient method of their choice (whether this be mental, written or in upper Key Stage 2 using a calculator) that is appropriate for a given task. They will do this by always asking themselves:**

**‘Can I do this in my head?’**

**‘Can I do this in my head using drawings or jottings?’**

**‘Do I need to use a pencil and paper procedure?’**

**‘Do I need a calculator?’**

Further supporting documents for the policy can be found on ‘MathsWeb’ ([www.leics.gov.uk/sips/lqfl\\_numeracy](http://www.leics.gov.uk/sips/lqfl_numeracy)).

- Homework help sheets
- Parents Leaflets containing ideas for fun maths activities
- Parent guidance on mental and written calculation (Sums and Things)
- Helping your child with maths
- Updating division - a paper discussing the progression in division
- Extra guidance for recording in mathematics for years R to 3.





# Addition

## Addition Objectives

### PF-calculating

Relate addition to counting on; recognise that addition can be done in any order; use practical and informal written methods to support the addition of a one-digit number or a multiple of 10 to a one-digit or two-digit number

*Use the vocabulary related to addition and subtraction and symbols to describe and record addition and subtraction number sentences*

### PF-knowing and using number facts

Derive and recall all pairs of numbers with a total of 10 and addition facts for totals to at least 5; work out the corresponding subtraction facts

Recording practical activities may be in the form of pictures or marks to support counting from more than one group.

## Addition Objectives

### PF-calculating

*Add or subtract mentally a one-digit number or a multiple of 10 to or from any two-digit number; use practical and informal written methods to add and subtract two-digit numbers*

Understand that subtraction is the inverse of addition and vice versa; use this to derive and record related addition and subtraction number sentences

*Use the symbols +, -, \*, ÷ and = to record and interpret number sentences involving all four operations; calculate the value of an unknown in a number sentence (e.g.  $\square \div 2 = 6$ ,  $30 - \square = 24$ )*

### PF-knowing and using number facts

Derive and recall all addition and subtraction facts for each number to at least 10, all pairs with totals to 20 and all pairs of multiples of 10 with totals up to 100

## Addition Objectives

### PF-calculating

Develop and use written methods to record, support or explain addition and subtraction of two-digit and three-digit numbers

*Add or subtract mentally combinations of one-digit and two-digit numbers*

### PF-knowing and using number facts

Derive and recall all addition and subtraction facts for each number to 20, sums and differences of multiples of 10 and number pairs that total 100



# Addition

## Addition Objectives

### PF-calculating

Refine and use efficient written methods to add and subtract two-digit and three-digit whole numbers and £.p

*Add or subtract mentally pairs of two-digit whole numbers (e.g.  $47 + 58$ ,  $91 - 35$ )*

## Addition Objectives

### PF-calculating

*Use efficient written methods to add and subtract whole numbers and decimals with up to two places*

## Addition Objectives

### PF-calculating

*Use efficient written methods to add and subtract integers and decimals, to multiply and divide integers and decimals by a one-digit integer, and to multiply two-digit and three-digit integers by a two-digit integer*

**+ = signs and missing numbers**

Continue using a range of equations as in Year 1 and 2 but with appropriate numbers.

**Partition into tens and ones and recombine**

Either partition both numbers and recombine or partition the second number only e.g.

$$\begin{aligned} 55 + 37 &= 55 + 30 + 7 \\ &= 85 + 7 \\ &= 92 \end{aligned}$$

**Add the nearest multiple of 10, then adjust**

Continue as in Year 2 and 3 but with appropriate numbers e.g. 63 + 29 is the same as 63 + 30 - 1

**Pencil and paper procedures**

$$367 + 185 = 431$$

**either**                      **or**

$$\begin{array}{r} 367 \\ +185 \\ \hline 12 \\ 140 \\ \hline 400 \\ 552 \end{array} \qquad \begin{array}{r} 300 + 60 + 7 \\ 100 + 80 + 5 \\ \hline 400 + 140 + 12 = 552 \end{array}$$

**leading to**

$$\begin{array}{r} 367 \\ +185 \\ \hline 552 \\ \hline 11 \end{array}$$

Extend to decimals in the context of money.

**+ = signs and missing numbers**

Continue using a range of equations as in Year 1 and 2 but with appropriate numbers.

**Partition into hundreds, tens and ones and recombine**

Either partition both numbers and recombine or partition the second number only e.g.

$$\begin{aligned} 358 + 73 &= 358 + 70 + 3 \\ &= 428 + 3 \\ &= 431 \end{aligned}$$

**Add or subtract the nearest multiple of 10 or 100, then adjust**

Continue as in Year 2, 3 and 4 but with appropriate numbers e.g. 458 + 79 = is the same as 458 + 80 - 1

**Pencil and paper procedures**

Extend to numbers with at least four digits

$$3587 + 675 = 4262$$

$$\begin{array}{r} 3587 \\ + 675 \\ \hline 4262 \\ \hline 111 \end{array}$$

Revert to expanded methods if the children experience any difficulty.

Extend to up to two places of decimals (same number of decimals places) and adding several numbers (with different numbers of digits).

$$\begin{array}{r} 72.8 \\ +54.6 \\ \hline 127.4 \\ \hline 11 \end{array}$$

**+ = signs and missing numbers**

Continue using a range of equations as in Year 1 and 2 but with appropriate numbers.

**Partition into hundreds, tens, ones and decimal fractions and recombine**

Either partition both numbers and recombine or partition the second number only e.g.

$$\begin{aligned} 35.8 + 7.3 &= 35.8 + 7 + 0.3 \\ &= 42.8 + 0.3 \\ &= 43.1 \end{aligned}$$

**Add the nearest multiple of 10, 100 or 1000, then adjust**

Continue as in Year 2, 3, 4 and 5 but with appropriate numbers including extending to adding 0.9, 1.9, 2.9 etc

**Pencil and paper procedures**

Extend to numbers with any number of digits and decimals with 1, 2 and/or 3 decimal places.

$$13.86 + 9.481 = 23.341$$

$$\begin{array}{r} 13.86 \\ + 9.481 \\ \hline 23.341 \\ \hline 111 \end{array}$$

Revert to expanded methods if the children experience any difficulty.

# Subtraction

## Subtraction Objectives

### PF-calculating

Understand subtraction as 'take away' and find a 'difference' by counting up; use practical and informal written methods to support the subtraction of a one-digit number from a one digit or two-digit number and a multiple of 10 from a two-digit number

*Use the vocabulary related to addition and subtraction and symbols to describe and record addition and subtraction number sentences*

### PF-knowing and using number facts

Derive and recall all pairs of numbers with a total of 10 and addition facts for totals to at least 5; work out the corresponding subtraction facts

## Subtraction Objectives

### PF-calculating

*Add or subtract mentally a one-digit number or a multiple of 10 to or from any two-digit number; use practical and informal written methods to add and subtract two-digit numbers*

Understand that subtraction is the inverse of addition and vice versa; use this to derive and record related addition and subtraction number sentences

*Use the symbols +, -, \*, ÷ and = to record and interpret number sentences involving all four operations; calculate the value of an unknown in a number sentence (e.g.  $\square \div 2 = 6$ ,  $30 - \square = 24$ )*

### PF-knowing and using number facts

Derive and recall all addition and subtraction facts for each number to at least 10, all pairs with totals to 20 and all pairs of multiples of 10 with totals up to 100

## Subtraction Objectives

### PF-calculating

Develop and use written methods to record, support or explain addition and subtraction of two-digit and three-digit numbers

*Add or subtract mentally combinations of one-digit and two-digit numbers*

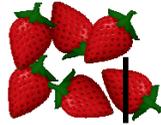
### PF-knowing and using number facts

Derive and recall all addition and subtraction facts for each number to 20, sums and differences of multiples of 10 and number pairs that total 100

**- = signs and missing numbers**

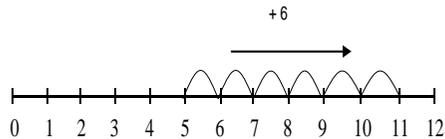
$$\begin{array}{ll} 7 - 3 = \square & \square = 7 - 3 \\ 7 - \square = 4 & 4 = \square - 3 \\ \square - 3 = 4 & 4 = 7 - \square \\ \square - \nabla = 4 & 4 = \square - \nabla \end{array}$$

- Understand subtraction as 'take away'



- Find a 'difference' by counting up;

I have saved 5p. The socks that I want to buy cost 11p. How much more do I need in order to buy the socks?



- Use practical and informal written methods to support the subtraction of a one-digit number from a one digit or two-digit number and a multiple of 10 from a two-digit number.

I have 11 toy cars. There are 5 cars too many to fit in the garage. How many cars fit in the garage?

Use the vocabulary related to addition and subtraction and symbols to describe and record addition and subtraction number sentences

Recording by

- drawing jumps on prepared lines
- constructing own lines

**- = signs and missing numbers**

Continue using a range of equations as in Year 1 but with appropriate numbers.

Extend to  $14 + 5 = 20 - \square$

**Find a small difference by counting up**

$$42 - 39 = 3$$

Subtract 9 or 11. Begin to add/subtract 19 or 21

$$35 - 9 = 26$$

**Use known number facts and place value to subtract**

(partition second number only)

$$\begin{aligned} 37 - 12 &= 37 - 10 - 2 \\ &= 27 - 2 \\ &= 25 \end{aligned}$$

**Bridge through 10 where necessary**

$$32 - 17$$

**- = signs and missing numbers**

Continue using a range of equations as in Year 1 and 2

but with appropriate numbers ■

Find a small difference by counting up

Continue as in Year 2 but with appropriate numbers

e.g.  $102 - 97 = 5$

Subtract mentally a 'near multiple of 10' to or from a two-digit number

Continue as in Year 2 but with appropriate numbers

e.g.  $78 - 49$  is the same as  $78 - 50 + 1$

Use known number facts and place value to subtract

Continue as in Year 2 but with appropriate numbers e.g.

$$97 - 15 = 72$$

With practice, children will need to record less information and decide whether to count back or forward. It is useful to ask children whether counting up or back is the more

# Subtraction

Subtraction Objectives

**PF-calculating**

Refine and use efficient written methods to add and subtract two-digit and three-digit whole numbers and £.p

*Add or subtract mentally pairs of two-digit whole numbers (e.g. 47 + 58, 91 - 35)*

**PF – Knowing & Using Number Facts**

Use knowledge of addition and subtraction facts and place value to derive sums and differences of pairs of multiples of 10, 100 or 1000

Subtraction Objectives

**PF-calculating**

*Use efficient written methods to add and subtract whole numbers and decimals with up to two places*

Extend mental-methods for whole-number calculations, for example to subtract one near-multiple of 1000 from another (e.g. 6070 - 4097)

**PF – Knowing & Using Number Facts**

*Use knowledge of place value and addition and subtraction of two-digit numbers to derive sums and differences of decimals (e.g. 6.5 ± 2.7, half of 5.6, double 0.34)*

Subtraction Objectives

**PF-calculating**

*Use efficient written methods to add and subtract integers and decimals, to multiply and divide integers and decimals by a one-digit integer, and to multiply two-digit and three-digit integers by a two-digit integer*

Calculate mentally with integers and decimals: U.t ± U.t,

**- = signs and missing numbers**

Continue using a range of equations as in Year 1 and 2 but with appropriate numbers.

Find a small difference by counting up

e.g.  $5003 - 4996 = 7$

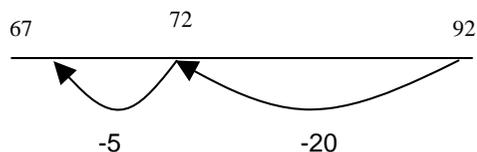
This can be modelled on an empty number line (see complementary addition below). Children should be encouraged to use known number facts to reduce the number of steps.

**Subtract the nearest multiple of 10, then adjust.**

Continue as in Year 2 and 3 but with appropriate numbers.

**Use known number facts and place value to subtract**

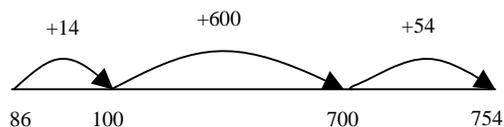
$92 - 25 = 67$



**Pencil and paper procedures**

Complementary addition

$754 - 86 = 668$



For those children with a secure mental image of the number line they could record the jumps only:

**$754 - 86 = 668$**

$$\begin{array}{r}
 14 \text{ (100)} \\
 600 \text{ (700)} \\
 \underline{54 \text{ (754)}} \\
 668
 \end{array}$$

**- = signs and missing numbers**

Continue using a range of equations as in Year 1 and 2 but with appropriate numbers.

Find a difference by counting up

e.g.  $8006 - 2993 = 5013$

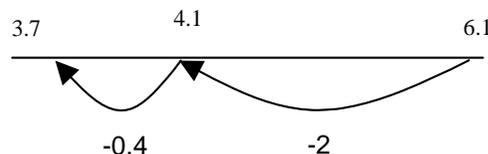
This can be modelled on an empty number line (see complementary addition below).

**Subtract the nearest multiple of 10 or 100, then adjust.**

Continue as in Year 2, 3 and 4 but with appropriate numbers.

**Use known number facts and place value to subtract**

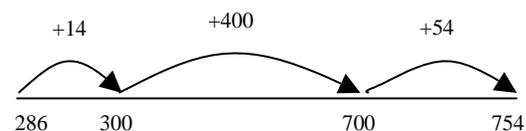
$6.1 - 2.4 = 3.7$



**Pencil and paper procedures**

Complementary addition

$754 - 286 = 468$



**OR**

$754 - 286 = 468$

|                 |                   |           |
|-----------------|-------------------|-----------|
| 14 (300)        | can be refined to | 14 (300)  |
| 400 (700)       |                   | 454 (754) |
| <u>54 (754)</u> |                   | 468       |
| 468             |                   |           |

Reduce the number of steps to make the calculation more efficient.

*Extend to 2 places of decimals*

**- = signs and missing numbers**

Continue using a range of equations as in Year 1 and 2 but with appropriate numbers.

Find a difference by counting up

e.g.  $8000 - 2785 = 5215$

To make this method more efficient, the number of steps should be reduced to a minimum through children knowing:

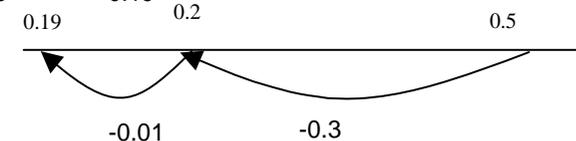
- Complements to 1, involving decimals to two decimal places (  $0.16 + 0.84$  )
- Complements to 10, 100 and 100

**Subtract the nearest multiple of 10, 100 or 1000, then adjust**

Continue as in Year 2, 3, 4 and 5 but with appropriate numbers.

**Use known number facts and place value to subtract**

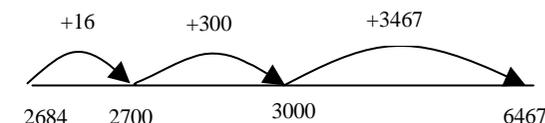
$0.5 - 0.31 = 0.19$



**Pencil and paper procedures**

Complementary addition

$6467 - 2684 = 3783$



**OR**

$6467 - 2684 = 3783$

|                    |                   |             |
|--------------------|-------------------|-------------|
| 16 (2700)          | can be refined to | 316 (3000)  |
| 300 (3000)         |                   | 3467 (6467) |
| <u>3467 (6467)</u> |                   | 3783        |
| 3783               |                   |             |

Reduce the number of steps to make the calculation more efficient.

*Extend to 2 places of decimals*

# Multiplication

## Multiplication Objectives

### **PF-calculating**

Solve practical problems that involve combining groups of 2, 5 or 10, or sharing into equal groups

### **PF-knowing & using number facts**

Count on or back in ones, twos, fives and tens and use this knowledge to derive the multiples of 2, 5 and 10 to the tenth multiple

## Multiplication Objectives

### **PF-calculating**

Represent repeated addition and arrays as multiplication, and sharing and repeated subtraction (grouping) as division; use practical and informal written methods and related vocabulary to support multiplication and division, including calculations with remainders

### **PF-knowing & using number facts**

Derive and recall multiplication facts for the 2, 5 and 10 times-tables and the related division facts; recognise multiples of 2, 5 and 10

## Multiplication Objectives

### **PF-calculating**

Use practical and informal written methods to multiply and divide two-digit numbers (e.g.  $13 \times 3$ ,  $50 \div 4$ ); round remainders up or down, depending on the context

Multiply one-digit and two-digit numbers by 10 or 100, and describe the effect

### **PF-knowing & using number facts**

Derive and recall multiplication facts for the 2, 3, 4, 5, 6 and 10 times-tables and the corresponding division facts; recognise multiples of 2, 5 or 10 up to 1000

Multiplication is related to doubling and counting groups of the same size.



Looking at columns  
 $2 + 2 + 2$   
 3 groups of 2

Looking at rows  
 $3 + 3$   
 2 groups of 3

**Counting using a variety of practical resources**

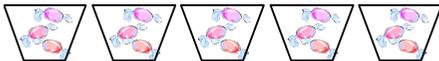
Counting in 2s e.g. counting socks, shoes, animal's legs...

Counting in 5s e.g. counting fingers, fingers in gloves, toes...

Counting in 10s e.g. fingers, toes...

**Pictures / marks**

There are 3 sweets in one bag.  
 How many sweets are there in 5 bags?



**x = signs and missing numbers**

$7 \times 2 = \square$                        $\square = 2 \times 7$   
 $7 \times \square = 14$                        $14 = \square \times 7$   
 $\square \times 2 = 14$                        $14 = 2 \times \square$   
 $\square \times \nabla = 14$                        $14 = \square \times \nabla$

**Arrays and repeated addition**


 $4 \times 2$  or  $4 + 4$   
 $2 \times 4$  or  $2 + 2 + 2 + 2$

**Doubling multiples of 5 up to 50**

$15 \times 2 = 30$

**Partition**

Children need to be secure with partitioning numbers into 10s and 1s and partitioning in different ways:  $6 = 5 + 1$  so e.g. Double 6 is the same as double five add double one.



AND double 15

OR

$$\begin{array}{r} X \quad 10 \quad 5 \\ 2 \quad 20 \quad 10 \quad = 30 \end{array}$$

**x = signs and missing numbers**

Continue using a range of equations as in Year 2 but with appropriate numbers.

**Arrays and repeated addition**

Continue to understand multiplication as repeated addition and continue to use arrays (as in Year 2).

**Doubling multiples of 5 up to 50**

$35 \times 2 = 70$

Partition

$$\begin{array}{r} X \quad 30 \quad 5 \\ 2 \quad 60 \quad 10 \quad = 70 \end{array}$$

**Use known facts and place value to carry out simple multiplications**

Use the same method as above (partitioning), e.g.  $32 \times 3 = 96$

$= 96$

# Multiplication

## Multiplication Objectives

### **PF-calculating**

*Develop and use written methods to record, support and explain multiplication and division of two-digit numbers by a one-digit number, including division with remainders (e.g.  $15 \times 9$ ,  $98 \div 6$ )*

Multiply and divide numbers to 1000 by 10 and then 100 (whole-number answers), understanding the effect; relate to scaling up or down

### **PF-Knowing and using number facts**

*Derive and recall multiplication facts up to  $10 \times 10$ , the corresponding division facts and multiples of numbers to 10 up to the tenth multiple*

## Multiplication Objectives

### **PF-calculating**

Refine and use efficient written methods to multiply and divide  $\text{HTU} \times \text{U}$ ,  $\text{TU} \times \text{TU}$ ,  $\text{U.t} \times \text{U}$  and  $\text{HTU} \div \text{U}$

### **PF-Knowing and using number facts**

Recall quickly multiplication facts up to  $10 \times 10$  and use them to multiply pairs of multiples of 10 and 100; derive quickly corresponding division facts

## Multiplication Objectives

### **PF-calculating**

*Use efficient written methods to add and subtract integers and decimals, to multiply and divide integers and decimals by a one-digit integer, and to multiply two-digit and three-digit integers by a two-digit integer*

### **PF-Knowing and using number facts**

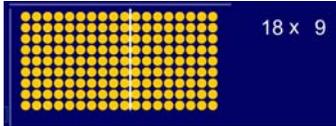
*Use knowledge of place value and multiplication facts to  $10 \times 10$  to derive related multiplication and division facts involving decimals (e.g.  $0.8 \times 7$ ,  $4.8 \div 6$ )*

**x = signs and missing numbers**

Continue using a range of equations as in Year 2 but with appropriate numbers

**Partition**

Continue to use arrays:



$$18 \times 9 = 162$$

$$18 \times 9 = (10 \times 9) + (8 \times 9) = 162$$

OR

Use the grid method of multiplication (as below)

**Pencil and paper procedures**

Grid method

23 x 7 is approximately 20 x 10 = 200

$$\begin{array}{r} x \quad 20 \quad 3 \\ 7 \quad 140 \quad 21 \quad = 161 \end{array}$$

**x = signs and missing numbers**

Continue using a range of equations as in Year 2 but with appropriate numbers

**Partition**

$$47 \times 6 = 282$$

$$47 \times 6 = (40 \times 6) + (7 \times 6) = 282$$

OR

Use the grid method of multiplication (as below)

**Pencil and paper procedures**

Grid method

72 x 38 is approximately 70 x 40 = 2800

Extend to simple decimals with one decimal place.

**x = signs and missing numbers**

Continue using a range of equations as in Year 2 but with appropriate numbers

**Partition**

$$87 \times 6 = 522$$

$$87 \times 6 = (80 \times 6) + (7 \times 6) = 522$$

OR

Use the grid method of multiplication (as below)

**Pencil and paper procedures**

Grid method

372 x 24 is approximately 400 x 20 = 8000

Extend to decimals with up to two decimal places.

# Division

## **PF - calculating**

Solve practical problems that involve combining groups of 2, 5 or 10, or sharing into equal groups

## **PF-knowing & using number facts**

Count on or back in ones, twos, fives and tens and use this knowledge to derive the multiples of 2, 5 and 10 to the tenth multiple

Division Objectives (excluding rapid recall)

## **PF - calculating**

Represent repeated addition and arrays as multiplication, **and sharing and repeated subtraction (grouping) as division**; use practical and informal written methods and related vocabulary to support multiplication and division, including calculations with remainders

## **PF-knowing & using number facts**

Derive and recall multiplication facts for the 2, 5 and 10 times-tables and the related division facts; recognise multiples of 2, 5 and 10

Division Objectives (excluding rapid recall)

## **PF - calculating**

Use practical and informal written methods to multiply and divide two-digit numbers (e.g.  $13 \times 3$ ,  $50 \div 4$ ); round remainders up or down, depending on the context

Understand that division is the inverse of multiplication and vice versa; use this to derive and record related multiplication and division number sentences

Find unit fractions of numbers and quantities (e.g.  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$  and  $\frac{1}{6}$  of 12 litres)

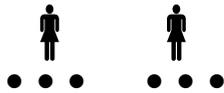
## **PF-knowing & using number facts**

Derive and recall multiplication facts for the 2, 3, 4, 5, 6 and 10 times-tables and the corresponding division facts; recognise multiples of 2, 5 or 10 up to 1000

### Sharing

Requires secure counting skills  
-see counting and understanding number strand  
Develops importance of one-to-one correspondence  
See appendix for additional information on x and ÷ and aspects of number

Sharing – 6 sweets are shared between 2 people. How many do they have each?



Practical activities involving sharing, distributing cards when playing a game, putting objects onto plates, into cups, hoops etc.

### Grouping

Sorting objects into 2s / 3s/ 4s etc  
How many pairs of socks are there?



There are 12 crocus bulbs. Plant 3 in each pot. How many pots are there?  
Jo has 12 Lego wheels. How many cars can she make?

### ÷ = signs and missing numbers

$$\begin{array}{ll} 6 \div 2 = \square & \square = 6 \div 2 \\ 6 \div \square = 3 & 3 = 6 \div \square \\ \square \div 2 = 3 & 3 = \square \div 2 \\ \square \div \nabla = 3 & 3 = \square \div \nabla \end{array}$$

### Grouping

Link to counting and understanding number strand  
Count up to 100 objects by grouping them and counting in tens, fives or twos;...

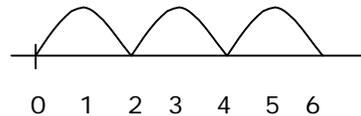
Find one half, one quarter and three quarters of shapes and sets of objects

6 ÷ 2 can be modelled as:

There are 6 strawberries.

How many people can have 2 each? How many 2s make 6?

6 ÷ 2 can be modelled as:



In the context of money count forwards and backwards using 2p, 5p and 10p coins

Practical grouping e.g. in PE

12 children get into teams of 4 to play a game. How many teams are there?



### ÷ = signs and missing numbers

Continue using a range of equations as in Year 2 but with appropriate numbers.

### Understand division as sharing and grouping

18 ÷ 3 can be modelled as:

Sharing – 18 shared between 3 (see Year 1 diagram)

**OR**

Grouping - How many 3's make 18?

### Remainders

$$16 \div 3 = 5 \text{ r}1$$

Sharing - 16 shared between 3, how many left over?

Grouping – How many 3's make 16, how many left over?  
e.g.

# Division

## Division Objectives

### PF – calculating

*Develop and use written methods to record, support and explain multiplication and **division of two-digit numbers by a one-digit number, including division with remainders (e.g.  $15 \times 9$ ,  $98 \div 6$ )***

### PF-Knowing and using number facts

*Derive and recall multiplication facts up to  $10 \times 10$ , the corresponding division facts and multiples of numbers to 10 up to the tenth multiple*

## Division Objectives

### PF – calculating

Refine and use efficient written methods to multiply and **divide**  
HTU  $\times$  U, TU  $\times$  TU, U.t  $\times$  U  
and HTU  $\div$  U

### PF-Knowing and using number facts

Recall quickly multiplication facts up to  $10 \times 10$  and use them to multiply pairs of multiples of 10 and 100; derive quickly corresponding division facts

## Division Objectives

### PF – calculating

*Use efficient written methods to add and subtract integers and decimals, to multiply and **divide integers and decimals by a one-digit integer**, and to multiply two-digit and three-digit integers by a two-digit integer*

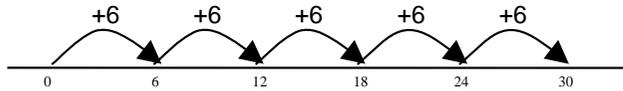
# Division

## ÷ = signs and missing numbers

Continue using a range of equations as in Year 2 but with appropriate numbers.

### Sharing and grouping

$30 \div 6$  can be modelled as:  
grouping – groups of 6 placed on no. line and the number of groups counted e.g.



sharing – sharing among 6, the number given to each person

Remainders  
 $41 \div 4 = 10 \text{ r}1$



$$41 = (10 \times 4) + 1$$

### Pencil and paper procedures

$72 \div 5$  lies between  $50 \div 5 = 10$  and  $100 \div 5 = 20$

\* Partition the dividend into multiples of the divisor:

e.g.  $72 = 50 + 22$

$$\begin{array}{r} 72 \\ \downarrow \downarrow \\ 50 + 22 \\ \downarrow \downarrow \\ 10 + 4 \text{ r}2 \end{array} \quad \text{OR} \quad \begin{array}{r} 72 \\ - 50 \quad (10 \text{ groups}) \\ \hline 22 \\ - 20 \quad (4 \text{ groups}) \\ \hline 2 \end{array}$$

Answer : 14 remainder 2

\* For a more detailed explanation see paper 'Updating division'.

## ÷ = signs and missing numbers

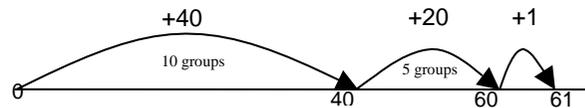
Continue using a range of equations as in Year 2 but with appropriate numbers.

### Sharing and grouping

Continue to understand division as both sharing and grouping (repeated subtraction).

## Remainders

Quotients expressed as fractions or decimal fractions  
 $61 \div 4 = 15 \frac{1}{4}$  or 15.25



### Pencil and paper procedures

$256 \div 7$  lies between  $210 \div 7 = 30$  and  $280 \div 7 = 40$

\* Partition the dividend into multiples of the divisor:

e.g.  $256 = 210 + 46$

$210 \div 7 = 30$

$46 \div 7 = 6 \text{ r}4 \rightarrow 30 + 6 \text{ r}4 = 36 \text{ r}4$

OR

$$\begin{array}{r} 256 \\ \downarrow \downarrow \\ 210 + 46 \\ \downarrow \downarrow \\ 30 + 6 \text{ r}4 \end{array} \quad \text{OR} \quad \begin{array}{r} 256 \\ - 210 \quad (30 \text{ groups}) \\ \hline 46 \\ - 42 \quad (6 \text{ groups}) \\ \hline 4 \end{array}$$

Answer: 36 remainder 4

\* For a more detailed explanation see paper 'Updating division'.

## ÷ = signs and missing numbers

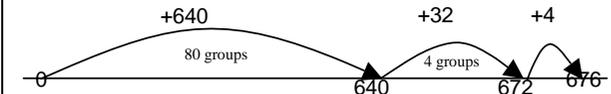
Continue using a range of equations as in Year 2 but with appropriate numbers.

### Sharing and grouping

Continue to understand division as both sharing and grouping (repeated subtraction).

## Remainders

Quotients expressed as fractions or decimal fractions  
 $676 \div 8 = 84.5$



### Pencil and paper procedures

$977 \div 36$  is approximately  $1000 \div 40 = 25$

\* Partition the dividend into multiples of the divisor:

e.g.  $977 = 720 + 180 + 77$

$720 \div 36 = 20$

$180 \div 36 = 5$

$77 \div 36 = 2 \text{ r}5 \rightarrow 20 + 5 + 2 \text{ r}5 = 27 \text{ r}5$

OR

$$\begin{array}{r} 977 \\ \downarrow \downarrow \downarrow \\ 720 + 180 + 77 \\ \downarrow \downarrow \downarrow \\ 20 + 5 \quad 2 \text{ r}5 \end{array} \quad \text{OR} \quad \begin{array}{r} 977 \\ - 720 \quad (20 \text{ groups}) \\ \hline 257 \\ - 180 \quad (5 \text{ groups}) \\ \hline 77 \\ - 72 \quad (2 \text{ groups}) \\ \hline 5 \end{array}$$

Answer:  $27 \frac{5}{36}$

\* For a more detailed explanation see paper 'Updating division'.

# Appendices

1. LEA – Key questions to consider
2. Updating Division discussion paper
3. Mental methods to enable chunking

# Appendix 1

LEA – Key questions to consider

## **Pencil and paper procedures KS1-3 LEA Recommendations**

### **Key questions to consider (Primary)**

“Schools are recognising, increasingly, the importance of adopting a common approach to the recording and layout of pupils’ work, but much remains to be done to put policies into practice.”

**Teaching of Calculation in Primary Schools, A Report by HMI, April 2002**

These recommendations have been written by the Leicestershire Numeracy Team to help schools begin to formulate their own policy. The guidance has been written only after acquiring a large evidence base from both primary and secondary children and in response to the findings from the above HMI report. It is from overwhelming evidence that children are not using decomposition successfully that we have chosen to emphasise complementary addition as a method for subtraction. Equally the successful use of the grid method of multiplication and ‘chunking’ as a method of division by children across the county has led the LEA to recommend their use.

**In this policy we recommend the route most children should be able to follow successfully and with understanding. However schools may wish to introduce other methods for more able pupils to investigate and explore.**

These recommendations **should not** be given to teachers and adopted until a whole school discussion has taken place. If, after in-depth discussion, schools choose not to follow these LEA recommendations, this policy will need to be adapted to reflect the decisions made. It is important to ensure that if any changes are made consistency and a clear progression is maintained.

We suggest schools spend some time at staff meetings to discuss this policy in detail. It is also important for maths coordinators to liaise with their High School and other feeder primary schools to ensure a clear progression into secondary education. We have written the following questions, which can be asked to promote discussion and help ensure that the resulting policy represents the methods of recording actually being used or adopted throughout the school.

## **General questions to consider**

- Do we have enough emphasis on the use of number lines throughout the school? Are they used to model all four operations?
- Are there methods of recording we don't use at the moment? Do we need to adopt them?
- Do children have the necessary mental calculations skills and strategies needed for the methods shown in each year group? If not these skills will need reinforcing before children are moved forward. Pages 6-11 of the QCA booklet 'Teaching mental calculation strategies'

provide a good overview of the mental strategies children need to develop each year and the mental calculations children should be able to perform.

- Are children encouraged to estimate the approximate size of a calculation and use this to check the ‘reasonableness’ of their answer?
- Are children given regular opportunities to use and apply calculation methods efficiently to solve a range of problems (including word problems)?

## Questions to consider when looking at addition

- Which expanded method for addition do we teach as a school?
- When is it appropriate for children to partition all numbers? (Children should generally be encouraged to only partition the second number. However when adding three or more numbers they may need to revert to partitioning all the numbers).

### Questions to consider when looking at subtraction

- How do we teach subtraction? Do we agree as a staff to teach complementary addition rather than teach decomposition?
- Is there enough emphasis on ‘difference’ (how many more/how many less) in Key Stage 1 or do children only view subtraction as ‘take away’?

# Questions to consider when looking at multiplication

- Do we currently teach the grid method of multiplication?
- Is there enough emphasis placed on arrays in Key Stage 1?
- Can the grid method be introduced as early as Year 2 when children partition to double numbers?

# Questions to consider when looking at division

- Is chunking used for division throughout Key Stage 2?
- Have empty number lines been used to demonstrate the idea of counting on in multiples of the divisor?

- Are the foundations of chunking taught in Years 2 and 3 (e.g. is there enough emphasis placed on grouping or are children only taught how to share?)?
- Is partitioning used regularly to support division?

## And finally...

- Is there whole school agreement on the methods in the policy? If not are there alternatives, which all staff agree on? How will consistency and progression be maintained?

Other useful sources of information:

- Teaching Mental Calculation Strategies (pages 6-11 give a clear overview of the mental calculation expectations for each year)
- Teaching written calculations (Part 2 explores the role of written calculations)

# Appendix 2

Updating Division discussion paper

## Updating division

This paper (extracts from 'Divide and Rule 2' from the NNS 3 + 2 course) looks at progression in division across Years 4, 5 and 6. It highlights some of the key knowledge and understanding children need to meet the age-related expectations set out in the Framework.

### Prerequisite skills for efficient mental multiplication and division

Before they develop further multiplication and division skills, pupils need to be able to do the following.

- Add and subtract accurately and efficiently.
- Recall quickly all the facts in the multiplication tables to  $10 \times 10$  and the division tables to  $100 \div 10$ .
- Understand the effect of multiplying and dividing whole numbers by 10 and 100 (and later 1000).
- Recognise factors of numbers (e.g. that  $15 = 5 \times 3$ , or that  $40 = 10 \times 4$ ).
- Derive other results from multiplication and division facts and multiplication or division by 10 or 100. For example, from  $4 \times 8 = 32$ , deduce that:

$$4 \times 80 = 320$$

$$4 = 320 \div 80$$

|                           |                         |
|---------------------------|-------------------------|
| $4 \times 800 = 3200$     | $4 = 3200 \div 800$     |
| $40 \times 8 = 320$       | $40 = 320 \div 8$       |
| $40 \times 80 = 3200$     | $40 = 3200 \div 80$     |
| $40 \times 800 = 32\,000$ | $40 = 32\,000 \div 800$ |
| $400 \times 8 = 3200$     | $400 = 3200 \div 8$     |
| $400 \times 80 = 32\,000$ | $400 = 32\,000 \div 80$ |

and so on. Notice how the inverse operation is being applied: if  $a \times b = c$ , then  $a = c \div b$ . (It helps to notice that the total number of zeros in the two numbers being multiplied is the same as the number of zeros after the 32 in the product.)

- Understand how the principles of the commutative, associative and distributive laws apply (or do not apply) to multiplication and division.

## The laws of arithmetic

Primary pupils don't need to know the names of these laws, or to see them expressed algebraically, but they do need to understand how the laws work in practice if they are to become proficient at multiplication and division.

*Commutative law of multiplication:*  $a \times b = b \times a$

*Associative law of multiplication:*  $(a \times b) \times c = a \times (b \times c)$

*Distributive law of multiplication:*  $(a + b) \times c = (a \times c) + (b \times c)$

Written formally, the laws of arithmetic can look awesome. But anyone who does a multiplication calculation will probably use the laws subconsciously.

For example, take the calculation  $5 \times 18$ . Most people prefer to think of this as 18 fives, rather than 5 eighTEENS, because the 5 times table is easier than the 18 times table. The *commutative law* of multiplication allows the order of the numbers to be switched as much as you wish. To do  $18 \times 5$ , you could think of 18 as  $9 \times 2$ , so the calculation becomes  $9 \times 2 \times 5$ . You could choose to do  $2 \times 5$  first, to get 10, then work out  $9 \times 10 = 90$ . This method uses the *associative law* of multiplication – the 2 is associated with the 5, rather than the 9, to make the calculation easier.

But you may prefer to calculate  $18 \times 5$  by splitting 18 into  $10 + 8$ , and multiplying the 10 and 8 separately by 5. This gives you 50 and 40, which add up to 90. This method uses the *distributive law* of multiplication. The multiplication by 5 is distributed across the addition of 10 and 8.

We also want pupils to realise that division:

*is not* commutative: for example,  $12 \div 3$  is not equal to  $3 \div 12$ ;

*is not* associative: for example,  $(24 \div 6) \div 2$  is not equal to  $24 \div (6 \div 2)$ , since  $(24 \div 6) \div 2 = 4 \div 2 = 2$ , and  $24 \div (6 \div 2) = 24 \div 3 = 8$ ;

but that *it can be* distributed across addition:

the *distributive law* of division;  $(a + b) \div c = (a \div c) + (b \div c)$

So provided that  $c$  is not zero (division by zero is not possible); if a dividend is the sum of two numbers  $(a + b)$ , you can divide each of  $a$  and  $b$  separately by the divisor  $c$  and add the results. For example:

$$96 \div 3 = (90 + 6) \div 3 = (90 \div 3) + (6 \div 3) = 30 + 2 = 32$$

Partitioning is useful in a division such as  $84 \div 7$ . Here the dividend of 84 is split not into  $80 + 4$  but into the greatest multiple of 7 tens or 70, plus the rest. So 84 is split into  $70 + 14$ , then each part is divided by 7.

$$\begin{array}{r} 84 \\ 70 + 14 \\ \downarrow \quad \downarrow \quad \div 7 \\ 10 + 2 = 12 \end{array}$$

$$\begin{aligned} 84 \div 7 &= (70 + 14) \div 7 \\ &= (70 \div 7) + (14 \div 7) \\ &= 10 + 2 \\ &= 12 \end{aligned}$$

Many pupils can apply the distributive law for multiplication with confidence. But this is not the case for the distributive law for division. One reason for this may be that the distributive law for division is not stated explicitly in the Framework and so may go unrecognised. Another may be that the teaching and learning of mental methods of division, stressing the correspondence to mental methods of multiplication, are not given enough attention to ensure that pupils' application of the distributive law for division becomes second nature.

## Moving towards a compact method of division

For division, as with multiplication, the vast majority of pupils in Years 4 to 6 should progress from informal pencil-and-paper methods to an efficient, compact written method. At the same time, they need to be reminded to approximate to gain a sense of the size of their answers.

For example, for  $81 \div 3$ , the dividend of 81 is split into the greatest multiple of 3 tens or 30, plus the rest. So 81 is split into  $60 + 21$ , and each part is divided by 3.

$$\begin{aligned}81 \div 3 &= (60 + 21) \div 3 \\ &= (60 \div 3) + (21 \div 3) \\ &= 20 + 7 \\ &= 27\end{aligned}$$

Division of three-digit numbers is done in stages. For  $672 \div 4$ , the dividend of 672 is first split into 400 (the greatest multiple of 4 hundreds or 400), plus the remaining 272.

$$\begin{aligned}672 \div 4 &= (400 + 272) \div 4 \\ &= (400 \div 4) + (272 \div 4) \\ &= 100 + (272 \div 4)\end{aligned}$$

Then the 272 is split into 240 (the greatest multiple of 4 tens or 40), plus the remaining 32.

$$\begin{aligned}272 \div 4 &= (240 + 32) \div 4 \\ &= (240 \div 4) + (32 \div 4) \\ &= 60 + (32 \div 4)\end{aligned}$$

Finally, the 32 is divided by 4, to give 8.

$$32 \div 4 = 8$$

so the 672 has been partitioned, in stages, to

$$\begin{aligned}672 \div 4 &= (400 \div 4) + (240 \div 4) + (32 \div 4) \\ &= 100 + 60 + 8\end{aligned}$$

$$= 168$$

We can use 'partitioning' to work out how much each person gets if £21.72 interest is shared between the 4 people who own the account.

$$\begin{aligned} 21.72 \div 4 &= (20 + 1.6 + 0.12) \div 4 \\ &= (20 \div 4) + (1.6 \div 4) + (0.12 \div 4) \\ &= 5 + 0.4 + 0.03 \\ &= 5.43 \end{aligned}$$

Similarly, we can work out how many packs of 24 we can make from 560 biscuits by repeatedly subtracting multiples of 24.

We might use:

$$\begin{aligned} 560 &= (480 + 72 + 8) \div 24 \\ &= (480 \div 24) + (72 \div 24) + 8 \text{ left that can't be grouped in 24s} \\ &= 20 + 3 + \text{remainder } 8 \end{aligned}$$

Pupils working confidently at level 4 at the end of Key Stage 2 should be tackling division of three-digit numbers using partitioning. Examples of the calculations that pupils at levels 4 and 5 need to experience are:

- no exchange, no remainder  $848 \div 4$  ( $800 + 40 + 8$ )
- no exchange, with remainder  $635 \div 3$  ( $600 + 30 + 5$ )
- with exchange, no remainder  $994 \div 7$  ( $700 + 280 + 14$ )
- with exchange, with remainder  $470 \div 3$  ( $300 + 270$ )
- first digit of the quotient not in most significant place  $287 \div 7$  ( $280 + 7$ )
- noughts in the quotient  $816 \div 4$  ( $800 + 16$ );  $5608 \div 8$  ( $5600 + 8$ )
- decimal dividend  $61.5 \div 5$  ( $50 + 10 + 1.5$ );  $4.26 \div 3$  ( $3 + 1.2 + 0.06$ )

## Appendix 3

Mental methods to enable chunking

# Mental methods children will need in order to divide using 'chunking'

|           |   |
|-----------|---|
| Reception | Count in tens.<br>Count in twos.  |
| Year 1    | Count on and back in tens from and back to zero.<br>Count on in twos from zero. |

|        |   |
|--------|---|
|        | <p>Count in steps of five from zero.</p> <p>Begin to count in steps of three from zero.</p> <p>Begin to know what each digit in a two-digit number represents. Partition a 'teens' number and begin to partition larger two-digit numbers into a multiple of ten and ones.</p>  |
| Year 2 | <p>Count in hundreds from and back to zero.</p> <p>Count on in twos from and back to zero.</p> <p>Count in steps of 3, 4 or 5 to at least 30, from and back to zero.</p> <p>Know what each digit in a two-digit number represents, and partition two-digit numbers into a multiple of ten and ones.</p> <p>Understand the operation of division as grouping (repeated subtraction) or as sharing equally.</p> <p>Interpret <math>8 \div 2</math> as 'how many 2s make 8?'</p> <p>Know by heart multiplication and associated division facts for the 2 and 10 times-tables and derive quickly the corresponding division facts.</p> <p>Use known number facts and place value to carry out mentally, simple divisions.</p> <p>Derive quickly halves of even numbers to 20 and begin to halve of multiples of 10 up to 100.</p>   |
| Year 3 | <p>Count on in steps of 3, 4 or 5 from any small number to at least 50 and then back again.</p> <p>Know what each digit represents, and partition three-digit numbers into a multiple of 100, a multiple of ten and ones.</p> <p>Know by heart multiplication and associated division facts for the 2, 3, 4, 5 and 10 times-tables.</p> <p>Know that dividing a whole number by 1 leaves the number unchanged.</p> <p>Begin to know the 3 and 4 times-tables.</p> <p>Understand that <math>16 \div 2</math> does not equal <math>2 \div 16</math>.</p> <p>Understand that division reverses multiplication and solve division calculations by using multiplication strategies.</p> <p>Understand the idea of a remainder.</p> <p>Make sensible decisions about rounding up or down after a division in the context of a problem.</p> <p>To divide by 10, shift the digits one place to the right.</p> <p>Use known number facts and place value to carry out mentally, simple divisions.</p> <p>Say a division statement corresponding to a given multiplication statement.</p> |
| Year 4 | <p>Begin to relate division and fractions.</p> <p>Partition numbers into thousands, hundreds, tens and ones.</p> <p>Partition &amp; use the distributive law (partition a number into multiples of the divisor), e.g. <math>36 \div 3 = (30 \div 3) + (6 \div 3)</math></p> <p>Multiply and divide any integer up to 1000 by 10 and understand the effect. Begin to divide by 100.</p>  |

|        |   |
|--------|---|
|        | <p>Know by heart multiplication and associated division facts for all times-tables.</p> <p>Use known facts and place value to multiply and divide integers, including by 10 and then 100.</p> <p>Use known facts and place value to add or subtract a pair of numbers mentally.</p> <p>Use doubling and halving, and factors.</p>   |
| Year 5 | <p>Relate division and fractions.</p> <p>Know what each digit represents in a number with up to two decimal places.</p> <p>Multiply and divide any positive integer up to 10 000 by 10 or 100 and understand the effect.</p> <p>Know by heart all multiplication and associated division facts up to 10 x 10.</p> <p>Use known facts and place value to add or subtract a pair of numbers mentally.</p> <p>Use known facts and place value to multiply and divide mentally.</p> <p>Partition &amp; use the distributive law (partition a number into multiples of the divisor), e.g. <math>75 \div 5 = (50 \div 5) + (25 \div 5)</math></p> |
| Year 6 | <p>Multiply and divide decimals mentally by 10 or 100, and integers by 1000, and explain the effect.</p> <p>Consolidate knowing by heart multiplication and associated division facts up to 10 x 10</p> <p>Partition &amp; use the distributive law (partition a number into multiples of the divisor), e.g. <math>98 \div 7 = (70 \div 7) + (28 \div 7)</math></p> <p>Use known number facts and place value to consolidate mental division.</p>   |

# Sums & Things ..

## Some help for parents

by the Leicestershire Numeracy Team

If you were brought up on pages of 'hard sums' you may think maths is difficult and boring. Worse than that you may think you're not very good at it!!! That's a real shame because it is a fun subject and for most people, if they are taught to understand numbers, they should be reasonably easy to grasp and use.

If what you remember as maths is pages of sums you may sometimes feel confused when your child's maths book contains writing, pictures, diagrams, jottings or blank number lines and not many 'formal calculations'. Certainly younger children, up to year 3, will record calculations in a variety of ways that do not necessarily look like the kind of 'sums' you remember. This is because written calculations are not the ultimate aim: the aim is for children to do calculations in their heads and, if the numbers are too large, to use a way of writing them down that helps their thinking.

As children develop their knowledge and understanding through years 3,4,5 and 6 teachers will be asking them to look at any calculation and ask "Can I do this in my head?" Sometimes this will need to be supported by a drawing, diagram or numerical jotting (notes). If they can't do it largely in their heads they should be looking for the most suitable written method or, during years 5 and 6, using a calculator for more complex calculations.

Here we try, as simply as possible, to help you to help your children. We take you through the ideas relating to children's number development from the earliest counting and mental skills to their recording of calculations to support thinking. If you've never felt very confident with numbers and calculations this also might help you, you never know! Read on and see.

# Looking at addition & subtraction

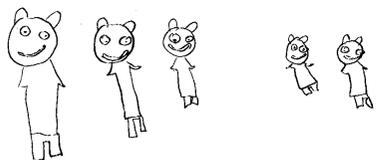
When children are in years 1 and 2 they are **not** expected to do vertical sums like

$$\begin{array}{r} 6 \\ +4 \\ \hline 10 \end{array}$$

but that doesn't mean they won't learn that  $6+4=10$ .

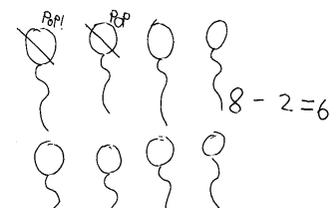
They will be using a daily mixture of practical, mental and oral work including lots of counting, talking about numbers and using numbers in real life activities. They will begin to record what they've done with pictures and numbers. These recordings will help them to understand what is happening and to show how they've worked something out. Here are two examples of early recording.

*Jane had 3 bears. She was given 2 more. How many does she have now?*



3     $3 + 2 = 5$     2

*There were 8 balloons. Two popped. How many are left?*



$8 - 2 = 6$



Children will also continue to use drawings, diagrams and blank number lines to support their thinking, as below.

*There are 13 boys and 8 girls in the room. How many altogether?*

13 + 8

*There are 34 children in the classroom. 27 go to the hall. How many are left?*

34 - 27:

Towards the end of the primary school years, children will be encouraged to use number lines to support their thinking, including vertical addition, using an ‘expanded method’, and subtraction using number lines by counting on (complementary addition). This form of recording will be used for those calculations that they can’t do ‘in their heads’. ‘Expanded methods’ are ways of recording that make the process of adding the different digits clear to children. These methods build on the mental methods they have been learning and should help children to understand what is happening. Here is an example of adding using an expanded method.

This is the same question as the one above, but using the expanded method of adding.

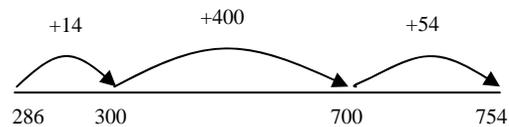
The diagram shows the same ideas. It includes an explanation of what might be said to describe the

|                                      |  |
|--------------------------------------|--|
| <p><b>The</b></p> <p><b>blue</b></p> | <p>Explanation.....</p> <p>The language used is very important to help children understand the size of numbers being added (e.g. <i>is it seven or seventy or seven hundred</i>)</p> <p><b>seven</b> plus <b>five</b> equals <b>twelve</b></p> <p><b>eighty</b> plus <b>forty</b> equals <b>one hundred and twenty</b></p> |
|--------------------------------------|--|

These methods mean that children may have to write a little more at this stage but, because it helps and supports their understanding, it enables them to become much more confident and quicker in the long run.

Here is an example of a subtraction problem involving 3 digit numbers:

$$754 - 286 = 468$$



**OR**

$$754 - 286 = 468$$

|   |                   |   |
|---|-------------------|---|
| $\begin{array}{r} 14 \text{ (300)} \\ 400 \text{ (700)} \\ \underline{54 \text{ (754)}} \\ 468 \end{array}$ | can be refined to | $\begin{array}{r} 14 \text{ (300)} \\ \underline{454 \text{ (754)}} \\ 468 \end{array}$ |
|---|-------------------|---|

Here you can see that children are using an image of a number line to support their thinking to work out this subtraction. Children are encouraged to find the difference between 286 and 754 by counting on, just as shopkeepers work out change, and how *you* might calculate mentally. Beginning at 286, 14 is needed to count on to the next 'friendly' number, 300; then a further 400 to land on 700; and finally another 54. All of the jumps total 468. Without the number line the calculation might look like the second example. This method of subtraction is known as 'complementary addition'.

Try to decide how you would do the following calculations. Would you do them in your head, write them down or use a calculator? The notes may change your mind but don't read them until you've had a go at the calculation.

|  |   |   |  |   |
|--|---|---|--|---|
| 45+99  | 3006-2999   | 2.3 + 6.99  | 4532-3768  | The total of all the numbers from 1 to 10   |
| This is easy to do if you think of 99 being one less than 100. So add 100 to make 145 and then take away 1 to give 144 | These numbers are very close on the number line. We need 1 to get from 2999 to 3000 and 6 more to get to 3006 so the difference is 7. So much easier than doing a vertical sum! | This addition involving decimals may look hard but it's easier if you think of it as money. £2.30 add £6.99. So add £7 to get £9.30 and take away 1p to get £9.29 Easy! | This, for most people, needs a pencil and paper or calculator if speed is important. | This is easy if you think of pairs of numbers making 10<br>$1 + 9 = 10$<br>$2 + 8 = 10$<br>$3 + 7 = 10$<br>$4 + 6 = 10$<br>Finally<br>$5 + 10 = 15$<br>giving a total of 55 |

It is important that calculations are presented in problems or horizontally, as above, to encourage children to think about the numbers as a whole, what they mean, what a sensible answer might be and the best method of working them out. It's all about giving children confidence with and control over numbers.

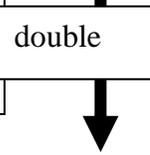
# Looking at multiplication & division

Did anyone ever tell you that you only needed to learn about half of the multiplication tables in order to know them all? If they didn't it was a bit mean because if you know  $3 \times 4 = 12$  you also know  $4 \times 3 = 12$ , so why learn it twice?

Did anyone ever say that once you knew the 2 times table, which is only double the 1 times, then the 4 times was easy because you just double the 2 times? Then you can double the 4 times to get the 8 times. The 3 times doubles to the 6 times and 12 times, the ten times can be halved to

give 5 times and so on. This uses the knowledge children are developing through addition and subtraction and makes important connections for them. This chart shows how this works for the 2x, 4x and 8x tables.

| x      | 1 | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 |
|--------|---|----|----|----|----|----|----|----|----|----|
| 2times | 2 | 4  | 6  | 8  | 10 | 12 | 14 | 16 | 18 | 20 |
| 4times | 4 | 8  | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 |
| 8times | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 |



It's also possible that you weren't told that you knew your division tables. If you were shown that division was the opposite of multiplication you will understand that knowing  $3 \times 4 = 12$  or  $4 \times 3 = 12$  also means you know  $12 \div 4 = 3$  and  $12 \div 3 = 4$ . So knowing one number fact, like  $3 \times 4 = 12$ , immediately means we know at least four.

But did you also realise that knowing any one of these facts helps you to know a lot more than four without actually learning them? Read on.

The early work children do will introduce them to the ideas of multiplication and division. They will be counting in different patterns, helped to see how multiplication is repeated addition and division is repeated subtraction, shown how division is the opposite of multiplication and taught to understand place value (that in 234 the 2 is 200, the 3 is 30 and the 4 is 4 ones (units)). This knowledge and understanding, with much of the work being done in their heads, opens up a whole world of facts for them and they don't all have to be memorised. That can make dealing with numbers feel a lot easier.

The following chart shows something of what this means.

**If you know  $4 \times 5 = 20$  what else do you know?**

$5 \times 4 = 20$

$20 \div 4 = 5$

$20 \div 5 = 4$

$4 + 4 + 4 + 4 + 4 = 20$

$5 + 5 + 5 + 5 = 20$

and using knowledge of place value

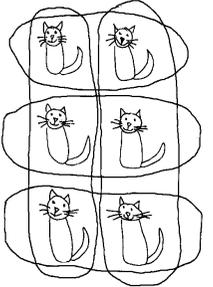
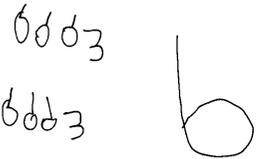
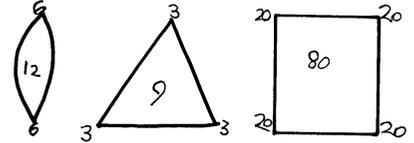
$5 \times 40 = 200$

$4 \times 50 = 200$

$200 \div 50 = 4$

The ability to do what you've just seen, developed gradually through years 1,2,3 & 4 helps children in years 4,5 & 6 to move on with confidence to multiplication and division of bigger numbers including those involving decimals and as with addition and subtraction, the questions will usually be presented to children as word problems or, horizontally, as calculations. The children should then be encouraged to work them out mentally if they can (supported by drawings, diagrams, number jottings if necessary) or, if they can't, to use the most suitable written method they know.

In years 1 & 2 the children will be recording to demonstrate how they have done something and to show that they've understood what is happening, as below.

|   |   |   |
|---|---|---|
|    |          |  <p data-bbox="1008 1364 1153 1460"> <math>12 = 2 \times 6</math><br/> <math>9 = 3 \times 3</math><br/> <math>80 = 4 \times 20</math> </p> <p data-bbox="996 1476 1220 1556"> <math>80 \div 4 = 20</math><br/> <math>80 \div 20 = 4</math> </p> |
| <p data-bbox="246 1468 571 1540"> <math>2 \times 3</math> cats = 6 cats or <math>3 \times 2</math> cats = 6 cats         </p> | <p data-bbox="616 1340 862 1412">           2 lots of 3 apples makes 6 apples.         </p> |   |

In years 3 & 4 the children will begin to use expanded methods to help them deal with calculations that they can't do in their heads. At this stage it will mostly involve multiplying and dividing 2 digit numbers by a single digit ( $72 \times 6$  or  $85 \div 4$ ). When dividing they will learn about and use remainders.

The expanded method for multiplying is often called the grid method. It uses the mental skills and the knowledge children have been learning and will help most children to move, with understanding, to the 'compact' method you may know.

This chart shows 'the grid method'. You can see, as with addition and subtraction expanded methods, it uses knowledge of number facts and the idea of splitting a number into its parts (place value) to help understanding of the process.

**How many sweets do I need for 24 party bags if each is to have 6 sweets?**

$$\begin{array}{r} x \quad 20 \quad 4 \\ 6 \quad \boxed{120} \quad \boxed{24} \quad = 144 \text{ sweets} \end{array}$$

You will see the 24 has been split into 20 and 4, each has been multiplied by 6 mentally and the two numbers added to give the final total. Many children will eventually develop the ability to do this kind of calculation totally in their heads.

Here is a slightly more difficult example.

**How much does it cost if I buy 9 books at 72p each?**

$$\begin{array}{r} x \quad 70 \quad 2 \\ 9 \quad \boxed{630} \quad \boxed{18} \quad = 648 \end{array}$$

It costs 648p or £6.48

The expanded method for division is often called 'chunking' and really just involves partitioning the number into helpful 'chunks' related to the number you are dividing by (divisor) or counting on/taking away chunks of the same size until you run out. It uses the fact that division is repeated subtraction of the same size group. So  $20 \div 4 = 5$  involves subtracting 4s from 20 until it's been used up or counting on in fours until you reach 20. You can do this 5 times.

Here is a more difficult example showing how larger chunks are used to speed up the process. Again this method uses and builds on the ideas explained earlier.

72 pears are packed in boxes of 6. How many boxes would there be?

First partition the 72 into chunks related to multiples of the divisor

$$72 = 60 + 12$$

then divide each part by 6

$$60 \div 6 = 10$$

$$12 \div 6 = 2$$

then add the 10 and the 2 to get 12

So 12 boxes of pears could be packed

As they notice  
this is not

$$10 + 2 = 12$$

is a suitable method of calculation, mentally if possible. Where  
or for more complex and involved work.

Most children will be expected to multiply 3 digits x 1 digit and 3 digits x 2 digits using a written method. Here are some examples.

**How many hours are there in the year 2003?**

This means we have to do  $365 \times 24$ ... a calculation you may find quite hard. Here it is expanded and using lots of mental skills... but none of them difficult.

|    |      |      |     |        |
|----|------|------|-----|--------|
| x  | 300  | 60   | 5   |        |
| 20 | 6000 | 1200 | 100 | = 7300 |
| 4  | 1200 | 240  | 20  | = 1460 |

So  $20 \times 365 = 7300$   
 $4 \times 365 = 1460$   
giving a total of 8760 hours with no difficult calculation to do.

More complex division will involve dividing 3 digit numbers by a 1 digit number and 3 digit numbers by a 2 digit number. With division, as with all calculation, it's important to think about what the actual problem is asking when you come to give an answer. This is shown up in the second example of expanded division or chunking involving buses for a school trip.

$458 \div 3$   
partition 458 into multiples of the divisor  
 $458 = 300 + 150 + 8$

$300 \div 3 = 100$   
 $150 \div 3 = 50$   
 $8 \div 3 = 2r2$   
add these together

$100 + 50 + 2r2 = 152$  remainder 2

458 stickers are shared between 3 children. How many does each get?

|              |                      |
|--------------|----------------------|
| 4 5 8        |                      |
| <u>3 0 0</u> | that's <b>100</b> x3 |
| 1 5 8        | left                 |
| <u>1 5 0</u> | that's <b>50</b> x3  |
| 8            | left                 |
| <u>6</u>     | that's <b>2</b> x3   |
| 2            | left over            |

So each gets  
 $458 \div 3 = 152$  remainder 2

432 children and adults are going on a school trip. If each bus takes 30 people how many are needed?

|   |   |   |                      |                    |
|---|---|---|----------------------|--------------------|
| 4 | 3 | 2 | people going         |                    |
| — | 3 | 0 | that's <b>10</b> x30 | or <b>10</b> buses |
|   | 1 | 3 | left                 |                    |
|   | — | 1 | that's <b>4</b> x30  | or <b>4</b> buses  |
|   |   | 1 | left                 | <b>1</b> bus       |

So we

$$432 \div 30 = 14 \text{ remainder } 12$$

This is not a good answer for this question because the 12 people left over would need another bus or they couldn't go!

So we see that 15 buses are needed...or some cars.

When the children  
though that the ex  
easier. They give

they are developed into a compact method. Remember  
the children feel more comfortable and therefore find it  
at what they are doing.

These methods are very useful when children are extending their work to numbers involving decimals but we'll leave that for another day!

Hopefully reading this has made you feel a little more confident and comfortable with numbers and calculations yourself, if you weren't already, and therefore better able to help your child. Remember if you have any concerns or questions it's always better to talk to your child's teacher

rather than pulling the children in different directions or worrying about what's going on. Children (and adults) need to feel confident with numbers and to enjoy playing with them and using them, that's really what it's all about. It then means using them for everyday purposes becomes a doddle rather than a threat.

Finally, have a go at these using expanded methods. Trying things out helps understanding and the answers are at the bottom of the box for you so you'll know if you are correct. One of them at least could be done completely in your head...maybe! Remember we want children to decide on the best method, mentally if possible, or the most suitable, manageable written method if not. Have fun.

|             |                  |                    |
|-------------|------------------|--------------------|
| calculation | a) $27 \times 7$ | b) $238 \times 45$ |
| working     |                  |                    |
| calculation | c) $176 \div 6$  | d) $346 \div 16$   |
| working     |                  |                    |

|         |       |            |         |             |
|---------|-------|------------|---------|-------------|
| answers | a)189 | c)29 rem 2 | b)10710 | d)21 rem 10 |
|---------|-------|------------|---------|-------------|

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Numeracy in Leicestershire 2005